# Atmospheric Transports and Energetics Subproject

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## 1.0 Introduction

The basic meridional transport of moisture, angular momentum and energy in the atmosphere requires a knowledge of the same basic statistics as does the Lorenz energy cycle. We propose to calculate and intercompare these very basic ways of characterizing the workings of the model atmospheres.

## 2.0 Transport equations

The vertically integrated and zonally averaged transport equation for a quantity *X* takes, to good approximation, the form

$$\frac{1}{a\cos\varphi}\frac{\partial}{\partial\varphi}F_{\varphi}\cos\varphi = [\bar{h}_T] - [\bar{h}_S] = S \qquad (1)$$

where

$$F_{\varphi} = F_m + F_s + F_t$$

$$= \int_o^{p_o}[\bar{\beta}\overline{Xv}]dp/g = \int[\bar{\beta}][\bar{X}]_R[\bar{v}]_R dp/g + \int[\bar{\beta}\bar{X}^*\bar{v}^*]dp/g + \int[\bar{\beta}\overline{X'v'}]dp/g \quad (2)$$

is the meridional flux of X which may be decomposed into components associated with the "mean meridional" circulation, the "standing eddies" and the "transient eddies" respectively. Here the formalism of Boer (1982) is followed where $\bar{\beta}$ is a variable which incorporates information on the whereabouts of the earth's surface in pressure coordinates so that various averaging and integration operations may be easily interchanged. The source/sink term S is the difference between $[\overline{h_T}]$ and $[\overline{h_S}]$, the vertical fluxes of *X* at the top of the atmosphere and at the surface.

It is often more useful to plot the associated *transports* defined as

$$\mathcal{T} = 2\pi a \cos\varphi F \qquad (3)$$

so that the pertinent equation is

$$\frac{1}{2\pi a^2}\frac{\partial \mathcal{T}}{\partial \varphi} = ([\bar{h}_T] - [\bar{h}_S])\cos\varphi = \mathcal{S}\cos\varphi \quad . \qquad (4)$$

The transport has the analogous components $\mathcal{T} = \mathcal{T}_m + \mathcal{T}_s + \mathcal{T}_t$ and may be evaluated from data. The total source/sink distributions may be decomposed into components associated with the respective transports $\mathcal{S} = \mathcal{S}_m + \mathcal{S}_s + \mathcal{S}_t$ by applying (4) to each transport component.

## 2.1 Basic budgets

The basic budgets of interest are those of moisture, angular momentum and energy:

**a. Moisture**

Here $X = q$ and

$$F_q = \int [\bar{\beta}\overline{qv}]dp/\mathrm{g} \quad \text{with } \mathcal{S}_q = [\bar{E}] - [\bar{P}]$$

**b. Angular momentum**

For angular momentum $M = a\cos\varphi(\Omega a\cos\varphi + u) = M_\Omega + M_r$ and, to good approximation, the equation for the relative angular momentum is

$$F_{AM} = a\cos\varphi\int[\bar{\beta}\overline{uv}]dp/\mathrm{g} \quad \text{with} \quad \mathcal{S}_{AM} = \tau_m - \tau_s$$

the "mountain" and "stress" torques $\tau_m = -\left[\frac{p_s}{g}\frac{\partial\Phi_s}{\partial\lambda}\right]$ and $\tau_s = a\cos\varphi[\tau_\lambda]$.

**c. Total energy (moist static energy)**

Here $X = C_pT + Lq + \phi$ and

$$F_H = \int[\bar{\beta}\overline{(C_pT + Lq + \phi)v}]dp/g \quad \text{with } S_H = [\overline{R_T}] - [\overline{R_S}] + [\bar{E}] + [\overline{H_S}]$$

the difference in radiative input at the top of the atmosphere and the surface, $E$ the evaporation and $H_s$ the sensible heat flux.

## 2.2 Calculations

There is a considerable condensation of the information by the zonal averaging and vertical integration so that all terms are one dimensional functions of latitude. The components of the transports $T = T_m + T_s + T_t$ and the associated source/sink terms $S\cos\varphi = (S_m + S_s + S_t)\cos\varphi$ would be calculated from (4), intercompared and displayed for all models. They would be compared with the results from ECMWF and NCEP reanalyses.

## 3.0 Lorenz energy budget

Terns in the Lorenz four-box energy cycle can be estimated to reasonable approximation with the same or similar statistics. The operative equations are

$$\frac{\partial A_Z}{\partial t} = -C(A_Z, K_Z) - C(A_Z, A_E) + G_Z \qquad \frac{\partial A_E}{\partial t} = -C(A_E, K_E) + C(A_Z, A_E) + G_E$$

$$\frac{\partial K_Z}{\partial t} = C(A_Z, K_Z) - C(K_Z, K_E) - D_Z \qquad \frac{\partial K_E}{\partial t} = C(A_E, K_E) + C(K_Z, K_E) - D_E$$

relating the zonal and eddy available potential and kinetic energy through generation, conversion, and dissipation processes.

The terms in these equations are commonly approximated by omitting terms involving $\omega$ in the conversion terms. In addition, the generation and dissipation terms are usually obtained as residuals. Here we propose to approximate the conversion terms in this way, but also propose to evaluate the important APE generation terms directly; this should be of interest in showing how the very basic generation rate of APE differs among models as well as giving a check on the consistency of the calculation. The dissipation terms would be obtained as residuals. For comparison purposes, the calculation of the terms in a consistent way is as least as important as the various approximations made. It is proposed to calculate the terms from the models and, in so far as possible, from the reanalysis data available at PCMDI and CCCma.

The proposed calculations are, for the amounts of APE and KE

$$A_Z = \frac{1}{2}C_p\int\gamma\overline{[\beta]}[\bar{T}]^{\dagger 2}dm \qquad A_E = \frac{1}{2}C_p\int\gamma[\bar{\beta}\overline{T_E^2}]dm$$

$$K_Z = \frac{1}{2}\int\overline{[\beta]}[\bar{\boldsymbol{V}}]_R\cdot[\bar{\boldsymbol{V}}]_R dm \qquad K_E = \frac{1}{2}\int[\bar{\beta}\overline{\boldsymbol{V}_E\cdot\boldsymbol{V}_E}]dm$$

for the approximate conversion terms

$$C(A_Z, K_Z) = -\int[\bar{\beta}][\bar{\omega}]^{\dagger}[\bar{\alpha}]^{\dagger}dm$$

$$C(A_E, K_E) = -\int[\bar{\beta}\overline{\omega_E\alpha_E}]dm$$

$$C(A_Z, A_E) \approx -C_p\int\left(\frac{\theta}{T}\right)\left([\bar{\beta}\overline{T_E v_E}]\frac{1}{a}\frac{\partial}{\partial\varphi}\left(\frac{T}{\theta}\right)\gamma[\bar{T}]^{\dagger}\right)dm$$

$$C(K_Z, K_E) \approx -\int a\cos\varphi\left(\frac{[\bar{\beta}\overline{u_E v_E}]}{a}\frac{\partial}{\partial\varphi}\left(\frac{[\bar{u}]_R}{a\cos\varphi}\right) + \left(\frac{[\bar{\beta}\overline{v_E^2}]}{a}\frac{\partial}{\partial\varphi} - \frac{\tan\varphi}{a}[\bar{\beta}\bar{V}_E\cdot\bar{V}_E]\right)\left(\frac{[\bar{v}]_R}{a\cos\varphi}\right)\right)dm$$

and, finally, the generation and dissipation terms

$$G_Z = \int\gamma[\bar{\beta}][\bar{T}]^{\dagger}[\bar{Q}]^{\dagger}dm \qquad G_E = \int\gamma[\bar{\beta}][\overline{T_E Q_E}]dm\,.$$

The dissipation terms $D_Z, D_E$ would be obtained as residuals.

Here the eddy terms $[\bar{\beta}\overline{X_E Y_E}] = [\bar{\beta}\bar{X}^*\bar{Y}^*] + [\bar{\beta}\overline{X'Y'}]$ include both the stationary and transient contributions which would, however, be evaluated separately for the components of APE and KE.

## 3.1 Data considerations

For the calculations proposed, the basic zonally averaged means and the standing eddy variance and covariance statistics

$$[\bar{X}]_R, [\bar{\beta}\bar{X}^{*2}], [\bar{\beta}\bar{X}^*\bar{v}^*] \quad \text{for} \quad X = u, v, \omega, T, q, \phi \tag{4}$$

are available from the quantities found in the proposed AMIP2 basic data set as listed in “Table 1, Upper-air low frequency (monthly mean) output”.

At the level of approximation proposed, the transient eddy variances

$$[\bar{\beta}\overline{X'^2}] \quad \text{for } X = u, v, T \tag{5}$$

and covariances

$$[\bar{\beta}\overline{X'v'}] \quad \text{for } X = u, \omega, T, q, \phi \tag{6}$$

are also needed and not all of them are available from “Table 1”, in particular, the temporal covariance $[\bar{\beta}\overline{\omega' T'}]$, which is basic to the conversion from APE to KE. It is very desirable also to retain the covariance $\overline{T'Q'}$ which would allow a direct calculation of eddy generation of APE.

While many of the terms in (4-6) are available in principle from “Table 1”, this proposal really only becomes feasible if the following suggestions can be accommodated.

(1) For the zonally and time averaged budgets calculations proposed here, *and because they are basic statistics of the flow*, the following quantities are *added* to Table 1;

variances $\overline{u'^2}, \overline{v'^2}, \overline{T'^2}$

covariances $\overline{v'\phi'}, \overline{\omega' T'}$ and, if possible, $\overline{T'Q'}$

while the eddy kinetic energy can be removed.

(2) To reduce the amount of data transfer required, the *zonally averaged* means, variances, and covariances in (4-6) be made available directly (rather than the full three-dimensional fields of “Table 1”). There are two choices here; one is that PCMDI calculates these statistics and makes them available (they would provide a useful quality control on the data); a second possibility is some arrangement whereby we calculate the statistics at PCMDI and transfer the resulting zonally averaged statistics to CCCma.

## 4.0 Summary

Means, variances and covariances

$$\bar{X}, \overline{X'^2}, \overline{X'\boldsymbol{V}'} \text{ for } X = u, v, \omega, T, q, \phi$$

of basic atmospheric variables arise naturally and physically in the *time-averaged* versions of the governing equations. These are three-dimensional fields and represent a considerable amount of data.

Under *zonal and time-averaging* the corresponding means, variances and standing and transient eddy covariances are

$$[\bar{X}]_R, [\bar{\beta}\bar{X}^{*2}], [\bar{\beta}\bar{X}^*\bar{v}^*], [\bar{\beta}\overline{X'^2}], [\bar{\beta}\overline{X'v'}] .$$

These are two-dimensional fields and represent a considerable reduction in the amount of data. The equations averaged in this way lead directly to the transport equations for moisture, momentum and energy and to the Lorenz energy cycle equations. It is proposed to evaluate and intercompare these basic physical processes as modelled by the AGCMs participating in AMIP2 and to compare the results with data from the ECMWF and NCEP reanalyses.

In order to accomplish this however, several *additional* statistics must be added to the proposed AMIP2 basic data set as listed in “Table 1, Upper-air low frequency (monthly mean) output” and the appropriate *zonally averaged basic statistics* made available.

## Reference

Boer, G.J., 1982: Diagnostic equations in isobaric coordinates. *Mon. Wea. Rev.*, **110**, 1801-1820.